A Look at Various Discounting Models

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Abstract

The standard discounting utility model from Samuelson has strong anomalies as shown by Loewenstein and Prelec. Hyperbolic discounting solves some of those anomalies but has its own flaws. We try to compare some discounting models as shown in [ObRo02], [LoTh92], [Ansl91] and [Rubi03].

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1 The Standard Discounted Utility Model

The standard discounted utility model from Samuelson, based on the preference of consumption today over consumption tomorrow, has the following form:

A utility function

$$U_t = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \cdots + \beta^T u(c_{t+T})$$

and a budget constraint

$$c_t + \frac{c_{t+1}}{1+r} + \frac{c_{t+2}}{(1+r)^2} + \cdots + \frac{c_{t+T}}{(1+r)^T} = y_t + \frac{y_{t+1}}{1+r} + \frac{y_{t+2}}{(1+r)^2} + \cdots + \frac{y_{t+T}}{(1+r)^T}$$

In the general literature like [ObRo02] the utility function is assumed to be logarithmic. Which is mostly a good enough assumption and keeps the math intuitive.

Maximizing the utility function under the budget constraint and the *no-Ponzi-game* condition in the case of $T \to \infty$. Leads to the *intertemporal Euler equation* where the consumer reaches an optimal distribution of consumptions over periods.

The model has dominated and strongly influenced the analyses and research of intertemporal choice. Over the years the discounted utility model has been applied to numerous topics like savings behaviour, labor supply, education decisions and crime. It has become a generally accepted framework for analyzing a broad field of economic decisions with delayed consequences.

The model is based on a exponential discount rate β^T.

But there are disadvantages of the original exponential discounted utility model, respectively function. The research of decision making under uncertainty has been also dominated by the findings and documentation of some assumed anomalies, i.e. behaviour that violate the expected utility axioms.

> Yet, in spite of its widespread use, the *discounted utility* model has not received substantial scrutiny in market contrast to the expected utility model for choice under uncertainty, which has been extensively criticized on empirical grounds and which has subsequently spawned a great number of variant models.
> [LoTh92]

2 Anomalies in the Discounted Utility Model

Loewenstein and Prelec named in [LoTh92] four major patterns that create difficulties for the discounted utility model.

2.1 The Common Difference Effect

We consider a person which is indifferent between adding x units to consumption at time t and $y > x$ units at a later time t, given a constant consumption level in all periods show, that preference between two consumption options depends only at the time interval separating them, so in our case t - t. For example, the preference between two delayed goods often changes when the delay of both goods is incremented by a constant amount. [Thal81] apple-example makes the point clear: a person might prefer an apple today to two apples tomorrow, but at the some time prefer two apples in 51 days to one apple in 50 days. Lwenstein and Prelec (1992) described this pattern as: the common difference effect.

2.2 The Absolute Magnitude Effect

Research results of time preference have also shown that large dollar amounts suffer less proportional discounting then smaller ones do. In his paper 1981, Thaler reported that individuals which were on average indifferent between getting $15 immediately and $60 in on e year, and were indifferent between an immediate $250 and $350 in one year too, or between $3000 now and $4000 in one year.

2.3 The Gain-Loss Asymmetry

A further finding is that losses are discounted at a lower rate than gains are. [Loew88c], for example, showed with his study, that subjects were mainly indifferent between receiving $10 immediately and receiving $21 in one year, and indifferent losing $10 immediately and losing $15 in one year. Thaler obtained in [Thal81] more dramatic loss-gain asymmetries, because they ware three to ten times greater than thus for losses. Recent research exhibited negative discounting where people preferred an immediate loss over a delayed loss of equal value.

2.4 Ad 2.2 and 2.3

The effects are problematic fort the discounted utility model in two ways: first, following [Mark52] then a pattern like described in the two effects above that results from people acting at one wealth level would create counterintuitive predictions for people at other wealth levels. And second, the baseline consumption level at the discounted utility model which is assumed

to be constant over the time periods is not constant with the above mentioned effects and anomalies.

2.5 The Delay-Speedup Asymmetry

A specific study by [Loew88a] described a forth anomaly, where between speeding up and delaying consumption exists an asymmetric preference. In principle, the amount which is required to compensate for delaying receiving a reward, by a given interval, from t to $t+s$, was from two to four times greater than the amount individuals were willing to give up to speed consumption up by the same interval from $t+s$ to t. After the two pairs of consumption are different representations of the same pair of options, the result leads to a framing effect which is incompatible with the discounted utility model and consequently the normative theory too.

3 *Rational* Examples of Hyperbolic Discounting

From George Ainslie in [Ansl91].

> Recent research has discovered frequent anomalies in the utilitarian reasoning of the normal human adult.
> [TvKa81]

One finding of the research is that the people preference for a smaller good often changes as a function of the time the choice is made, even though the difference in delay stays constant. This result is inimical to the rational market. A short example will underpin the above described finding. Fact is that a great majority of adults report that they would rather have \$50 immediately than \$100 in 2 years, but nearly no one prefers \$50 in 4 years over \$100 in 6 years, even though this is the same choice seen at 4 years' greater distance.

Following the conventional utility theory, the value of delayed goods is discounted in an exponential curve where the curves from two alternative amounts of the same good available at different times should never cross without any new information. But, based on studies of choice conducted by behavioural psychologists, a radically different discount function, known as Herrnstein's matching law, has become well known. Highly different experimental designs have given the curve al lot of individual forms, but all have

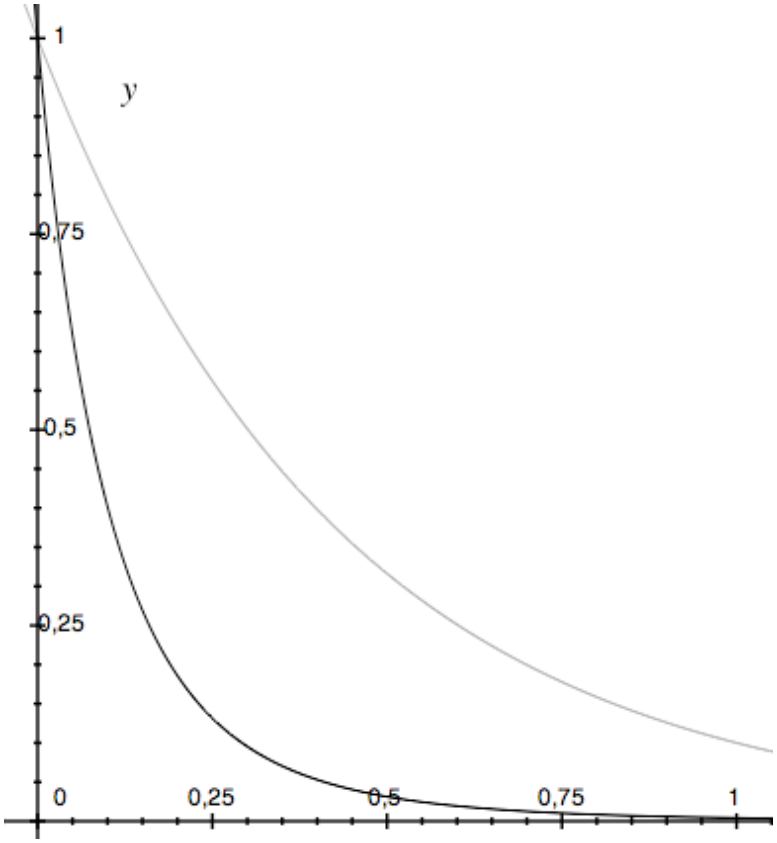

Figure 1: Exponential (gray) and hyperbolic (black) discount rate over time

been hyperbolic. The most useful version of an hyperbolic discount curve is possibly from Mazur in [MaNR87]

$$V = \frac{A}{\zeta + \Gamma(T - t)}$$

V is the good's value in competing with alternatives. A is the amount of the good, T the time at which each good is available, and t the time of the behaviour that obtains it. So $T - t$ describes the delay from the moment of choice. ζ is an empirical constant and represents the value of the good at zero delay, where Γ modifies the steepness of the delay gradient and is an empirical constant too.

Now Mazur's formula is used to compare two goods separated by three units of time. In addition, the later good is twice as large as the earlier one. So in this case the indifference point, where $V_{later} = V_{earlier}$, occurs when the earlier good is available two units of time from the moment of choice. When the delay is five units, the larger good is preferred by $\frac{2}{9}$ to $\frac{1}{6}$, while with a

delay of only one unit the larger good loses by $\frac{2}{5}$ to $\frac{1}{2}$. This example clearly shows that the smaller good is temporarily preferred to the larger when the delay is short.

As long as the choice is governed by the matching law, the tendency to form temporary preferences will present a major problem to rational planning.

> Any plan requiring a prolonged course of action will fail unless the person can arrange consistent motivation for a binding commitment to it.
> [Ansl91]

Looking at the long view of a person, he may want to be generally thin but to accomplish this goal he will have to overcome strong desires for food in the immediate future.

But also financial markets display a different pattern of human choice making. Participants behave as though they discount future goods at single digits and exponent rates in a very constant way. Now there is a major question, which is the true discount function, the matching law or the exponential rate from the bank? For [Ansl91] is the discount function, when he argues as follows, some kind of mixture:

> A careful look at the implications of temporary preference formation suggests mechanics by which a person who evaluates goods strictly according to the matching law can be expected to arrive at the bankers shallow exponential discount curves for at least some of his transactions, but imperfectly, as a result of varying skill an effort.

3.1 Stability and Intertemporal Prisoners Dilemmas

The in the quote above named mechanism is a kind of a personal rule, which can be derived from the matching law as follows: When a person must make as series choices between goods of amount A_i and later, larger goods of amount $A`_i$, so that all $A`_i > A_i$ and all $T`_i > T_i$, each choice will be described simply by the matching law evaluation of the two alternatives where the choice will be partly determined by the summed values of the goods on each side. Based on formula (1) the crucial time at which preference between the two whole series of goods changes will be represented by the t when the value $V`$ of the series of larger goods equals the value V of the series of smaller ones, called T_{indiff} in formula 1:

$$\frac{V}{V`} = \frac{\sum A_i/\zeta + \Gamma(T_i - t_{indif})}{\sum A`_i/\zeta + \Gamma(T`_i - t_{indif})} = 1$$

As a result, if the choice is made before t_{indiff}, it will favour the series of larger goods, and if it is made after tindiff it will favour the series of smaller, earlier ones. This sounds trivial, except for an important phenomenon:

> t_{indiff} between the series of larger goods and the series of smaller ones will move closer to the moment when the first smaller good is available as the series is made longer.
> [Ansl75]

The effect of choosing a whole series of goods at once is to increase the individuals tendency to choose the larger goods. But how can a person arrange to choose whole series of goods at once? Answer: The persons best information is his knowledge of his past behaviour, in other words his experience acting under similar circumstances which the most matching examples being the most informative. When the person has chosen both, the smaller and the larger good often enough, the choice will represent the swing of his expectation of future goods one way or the other. He will not necessarily notice the process or develop any way of describing it, but when he has become recognised this phenomenon, he will be able to induce it spontaneously and learn by the trial and error principle.

According to [Ansl91], this kind of rules can be seen as a solution to a bargaining problem. The temporal preference phenomenon produces a relationship with regard to the individuals successive motivational states. Successive motivational states can be divided in first, common ones and second, others that are peculiar or strange to them. The interests in common are similar with the persons long-range interests, whereas the peculiar ones are short-range interests.

To make the phenomenon clear we will take a look at two real-life-examples.

1. An alcoholic want to drink less in the long-term respectively aggregate way because he does not want to be an alcoholic, but he may want to drink a great amount of alcohol now. In other words, his long-term interest, common to all his successive motivational states, is to be generally sober. This interest is strongly challenged, and often overwhelmed, by a succession of short-range interests in getting drunk just once.

2. Imagine that a person at midnight time faces the choice of staying up for two more hours and having fun before he finally gets to bed, but feeling tired at work the whole next day, versus giving up his present fun and expecting to feel rested at work. The person values the additional fun at 60 units per hour and expects to lose 60 units per hour of comfort when he gets up at 7 a.m. until he leaves work at 5 p.m. So at midnight,the value of staying up is:

$$V_{up} = \sum_{i=0.5 \rightarrow 1.5} 60/(1+i) = 64$$

and the value of feeling rested at work:

$$V_{bed} = \sum_{i=7.5 \rightarrow 16.5} 60/(1+i) = 49$$

As the results make clear ($64 > 49$), he will stay up and suffer the next day. When the person faces the choice not only once but nightly, he may perceive his current choice as a precedent for future nights as well. Consider he believes that he will go to bed 2 hours later on 10 subsequent nights, the values of this alternatives are:

$$\begin{aligned} V_{up} \;=\; & \Big(\sum_{i=0.5 \rightarrow 1.5} 60/(1+i)\Big) \\ & + \Big(\sum_{i=24.5 \rightarrow 25.5} 60/(1+i)\Big) \\ & + \ldots \Big(\sum_{i=216.5 \rightarrow 217.5} 60/(1+i)\Big) = 78 \end{aligned}$$

And in the other hand 105 for going to bed early on the next 10 nights (same calculation). Taking the two results of his first choice and the present values of two subsequent series of 9 choices, 14 for always staying up and 56 for always going to bed, his incentives creates a prisoners dilemma between his own present and future motivational states (Table1), which he will face nightly. If the person decides to stay up today and go to bed on time in the future, that is worth 120. In contrast, if he goes to bed today but do not so in the future, that is worth only 65. So the best decision for the person will mainly depend in which way he forecasts his future perceptions. Hence, so long as he sees his current choice as a precedent, he will face the incentive of a repeated prisoners dilemma.

	Future	
Now	Stay Up	To Bed
Stay up	$64 + 14 = 78$	64+56=120
To bed	$49 + 14 = 63$	49+56=105

Table 1: Motivational states

3.2 A Persons Valuation of Money

Valuations of financial transactions are very similar to the valuations of the visceral goods like the above treated alcohol and sleep. For example, a person likes to skimobile in the winter and sail in the summer, and he is willing to pay what each equipment costs ($1000 for a used model on which there is little annual depreciation) at the beginning of the season. The person is able to sell each of the two equipments back to a store at the end of the season, getting 25 cents per dollar. Imagine that at every 6 month, he has to choose between getting $250 immediately and saving $1000 in 9 months. This alternatives evaluates formula (1), when $\zeta = 1$ month, at $250 and $100. Based on this evaluation the matching law predicts that he will sell. Under different circumstances, for example when he faces a similar choice twice a year, the incentives he faces are the sum of 40 times $250, one immediate, the other discounted, versus 40 times $1000, each 9 months after the corresponding $250. As a result, also these possibilities represent a prisoners dilemma (Table2).

	Future	
Now	Sell	Hold
Sell	$250 + 167 = 417$	250+489=739
Hold	$100 + 167 = 267$	100+489=589

Table 2: Value of contingencies

Following [Ansl91], there are three factors which help to reach some stability in this thematic. First, in principle cash prising makes a wide range of transactions easily comparable and leads to an encompassing personal rule about the value of money in general. People become more constant in choosing between related contingencies because of seeing his choices. Following this conclusion, if he sees what he spends for food or clothes etc. as examples of wasting or not wasting money, he will add thousands to his interdependent set of choices, each flattening his effective discount curve by a greater or lesser amount. Further can be argued that emotional comfort today is more important than the next years one. The resulting stability is

intensive to cost effectiveness.

Second, transactions in the finance sector are very often rivalrous ones. This adds an additional bonus to the intrinsic consumption value of goods involved in these transactions. Because of that, in lying and selling, he is not choosing simply in parallel with his neighbours, but in competition with them. Such a competition can be also found in (where the society is responsibly) nonfinancial sectors. For instance, people gain advantage by staying hungry to obtain the state-of-art modern stylish slimness. The personal rules governing these activities gain power from these additional value, better said stakes, just as the rules governing the value of money do. So we can argue that as the cash pricing labels many of the personal choices as comparable, so it engages the largest number of people in social competition.

And third, a person can manage his personal rules in a way that investment decisions are not weighted against his strongest temptations. [ShTh88] pointed out, that people assign their wealth to different mental accounts such as current and future income and current assets.

> These accounts seem to represent personal rules for how readily the money they govern may be used to satisfy immediate wants. (Ainslie, 1991)

Finally, the by George Ainslie discussed mechanisms in generally able to describe why priceable goods are not usually evaluated simply following the matching law, even though that law may underpin all valuations. Ainslie described further, that goods represent assigned values based on their strategic importance nested in a repetitive intertemporal prisoners dilemma where the goal is to maximise objective income despite of the influence of spontaneous preferences.

> The consequence for utility theory is that most valuations are mode not only or even mainly according to a present hunger, but according to toe precedents they will set. If such a valuation process can sometimes make five pounds sterling worth more than ten, it may have the power to account for a number of other anomalies in utility theory as well.
> [Ansl91]

4 Flaws in Hyperbolic Discounting

In [Rubi03], Rubinstein refers to Brocas and Carrilo [BrCa99] who state that:

> There is a well documented literature both in psychology and more recent in economocisc showing that individuals' discount rates are best approximated by hyperbolic rather than the traditional exponential functions. (There is) empirical support of this theory both in animals and humans...

Rubinstein quickly neglects' those results as not very convincing and shows his preference for a similarity-based procedural approach.

Rubinsteins' procedural approach: In his present discussion, the objects of choice are of the form (x, t) whereby x is an *amount* received with a *delay* of t units of time. Rubinstein thinks that when comparing two pairs (x, t) and (y, s), many decision makers go through a three-stage procedure using two similarity relations, one in the money dimensions and one in the time dimension:

Stage 1 The decision maker looks for dominance: If $x > y$ and $t < s$ then pair (x, t) is preferred over (y, s).

Stage 2 The decision maker looks for similarities between x and y and between t and s. If he finds similarity in one dimension only, he determines his preference between the two pairs using the dimension in which there is no similarity. For example, if t is similar to s but x is not similar to y, and $x > y$, then (x, t) is preferred over (y, s).

Stage 3 If the first two stages were not decisive the choice is made using a different criterion.

Rubenstein then shows findings from various experiments proving the dominance of his similarities-based procedural approach.

5 Conclusion

Exponential discounting my seem intuitive (one the first sight at least), but hyperbolic discounting lines up better with experimental data from psychology. Rubinstein's approach does improve the hyperbolic approach for certain cases, but it comes with high costs. Giving up transitivity and linearity, looks more like an revolution then evolution in utility modeling.

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