

# Human Migration and Migration of Firms An Agent based Simulation

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## Abstract

Workers, considering migration costs, will change location if expected wages are higher. The same counts for firms. Trying to increase profits, firms switch location if they expect wages to be lower.

I try to simulate both, human migration and migration of firms. Each influencing wages in their respective location. It is a inductive behavior model with various strategies. Just like the El Farol Problem [Arth94] except that it has more than one location.

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# 1 Introduction

This simulation is based on interacting *agents*. Agents are either *workers* or *firms*.

Each agent decides independent if and where to move. Trying to max his lifetime income. There is no communication between agents and the only information available is past wages at various locations.

As will be formalize later, the wage in country  $x$  is given by  $w_x = F_x/N_x$ , yielding a negative relation to the number of people  $N_x$  in  $x$  and a positive relation to the number of firms  $F_x$  in  $x$ . More workers means more competition among them. Forcing them to accept lower wages. The reverse is true for firms. More firms, competing for a constant number of workers, lift wages.

The behavior of workers and firms in this problem can be modeled as inductive behavior. The *El Farol Problem* provides the foundation.

We can adapting the El Farol Problem for the case of migrating workers and firms. Following Arthur:

First, if there were an obvious model that all agents could use to forecast (wages) and base their decisions on, then a deductive solution would be possible. But this is not the case here. Given (wages) in the recent past, a large number of expectational models might be reasonable and defensible. Thus, not knowing which model other agents might choose, a reference agent cannot choose his in a well-defined way. There is no deductively rational solution - no correct expectational model. From the agents viewpoint, the problem is ill-defined and they are propelled into a world of induction. Second, and diabolically, any commonalty of expectations gets broken up: If all workers believe *few* will go to  $B$  (therefor yielding higher wages in  $B$ ), all will go to  $B$  driving wages *down*. But this would invalidate that belief. Similarly, if all believe *most* will go, *nobody* will go, invalidating that belief. Expectations will be forced to differ.

The El Farol Problem provides the starting point for this simulation. In the following sections I describe how workers and firms form their wage expectations.

## 2 Workers following higher Wages

A worker will move from  $A$  to  $B$  if his expected lifetime income in location  $B$  minus his migration costs  $c_{A \rightarrow B}$  is greater than his expected lifetime income.

$$A \xrightarrow{c_{A \rightarrow B}} B$$

$$E(\Phi(B)) - c_{A \rightarrow B} > E(\Phi(A)) \quad (1)$$

Migration costs  $c_{A \rightarrow B}$  capture every influence on the decision to move from one location to another. Another location may be another village, state, country or continent. This costs include direct costs like transportation, immigration and working visa as well as indirect cost (costs of emigrating) like leaving family and friends behind or the life with a new culture and a new language. One may even have to pay people-smugglers for emigration from  $A$  (North Korea) and immigration to  $B$  (a first world country). All this adding up to the costs of migration.

Obviously this costs differ a lot. Migrating from Salzburg to Vienna or from Pyongyang to Washington D.C. certainly makes a difference. For workers *and* firms. For more details and a good comparison of direct and indirect costs of human migration consult Sjaastad's [Sjaa62].

In case of migration from  $A$  to  $B$  via  $Z$  costs of migration just add up.

$$A \xrightarrow{c_{A \rightarrow Z}} Z \xrightarrow{c_{Z \rightarrow B}} B = A \xrightarrow{c_{A \rightarrow Z} + c_{Z \rightarrow B}} B$$

In this simulation migration cost must be payed by the agents savings  $S$ . Workers and firms saving is a stock. Saving equals the sum of past income minus past expenses in form of migration costs.

Therefor migration is not possible if (2) is not satisfied.

$$c_{A \rightarrow B} < S \quad (2)$$

Abstracting from any consumption may seem harsh but consumption is not key for this simulation and instead of arbitrarily assuming some consumption, fixed or as percentage of income, we can as well just leave it out.

Finally the lifetime income  $\Phi$  is the sum of all discounted future wages. For

location  $L$  the expected<sup>1</sup> lifetime income for a worker is

$$E(\Phi(L)) = \sum_{n=0}^{\infty} \beta^n E(w_{t+n,L}) \quad (3)$$

### 3 Firms following lower Wages

A firm, following the same rationality as a worker, tries to maximize his expected lifetime income. Therefore migration from  $A$  to  $B$  will maximize  $E(\Phi)$  if (1) and (2) hold.

Migration cost  $c_{A \rightarrow B}$  for firms are higher than migration cost for workers but never the less follow the same logic. Capturing distance and differences in (work) culture, language, government regulations, accounting rules, taxes and so on. Again collected in  $c_{A \rightarrow B}$ .

Profit  $\pi$  is given by a simple production function with decreasing return of scale for labor  $n$  (the number of worker as the only input) and the cost of labor  $nw$ .

$$\pi = \text{Log}(n) - nw \quad (4)$$

Therefore the expected lifetime income of a firm is:

$$E(\Phi) = \sum_{s=0}^{\infty} \beta^s (\text{Log}(n_{t+s}) - n_{t+s}w_{t+s}) \quad (5)$$

Maximizing profit  $\pi$  for the number of employees  $n$  in (4) yields  $w = 1/n$ . This result connects the movement of workers with the movement of firms. Both influencing wages.

All firms share the same production function (4). Therefore wages payed in location  $x$  are equal for all agents in  $x$  given by (6) as stated in the introduction.

$$w_x(N_x, F_x) = \frac{F_x}{N_x} \quad (6)$$

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<sup>1</sup>Expectation:  $E_{t+1}(a_t) = E_t(a_t) = E(a_t)$  but  $E(a_t) \neq E(a_{t+1})$

## 4 Wage Expectations

Agents form their wage expectations based on past wages as collected in matrix  $W$ .

$$W = \begin{pmatrix} w_{1,t} & w_{2,t} & \dots & w_{L,t} \\ w_{1,t-1} & w_{2,t-1} & \dots & w_{L,t-1} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1,0} & w_{2,0} & \dots & w_{L,0} \end{pmatrix}$$

Agents have access to certain parts of matrix  $W$  depending on their wage predictor. Wage predictors are randomly assigned to each agent. In the following list  $E(w_{x,t})$  is the expected wage for location  $x$ .

### Wage Predictors

- the same as last week:  $E(w_{x,t}) = w_{x,t-1}$
- the average of the last three weeks:  $E(w_{x,t}) = [w_{x,t-1} + w_{x,t-2} + w_{x,t-3}] / 3$
- the same as two weeks ago:  $E(w_{x,t}) = w_{x,t-2}$
- the average of all last week wages:  $E(w_{x,t}) = \sum_{i=0}^C w_{i,t-1} / C$  where  $C$  is the number of different locations
- the inverse of last week:  $E(w_{x,t}) = -w_{x,t-1}$
- random walk<sup>2</sup>:  $E(w_{x,t}) = \text{randomize}(w_{x,t-1})$
- ...

Each worker will pick the location with the highest (expected) wage  $L^* = \text{Max}(E(w_{1,t+1}), E(w_{2,t+1}), \dots, E(w_{L,t+1}))$ .

Firms will decide analog but pick the location with the lowest (expected) wage  $L^* = \text{Min}(E(w_{1,t+1}), E(w_{2,t+1}), \dots, E(w_{L,t+1}))$ .

Agents compare the expected lifetime income in  $L^*$  with the expected income in the current location if migration can improve their income  $L^c$  (1) and they can afford to migrate (2) they will move from  $L^c$  to  $L^*$ .

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<sup>2</sup>Random walk, i.e. an agent randomly picking locations, will be an important control group. Successful predictors should out-perform random walk.

$$L^c \xrightarrow{C_{L^c \rightarrow L^*}} L^*$$

## 5 Simulation

Using the *Repast Agent Simulation Toolkit* I wrote a simple prototype capturing the above model.

Following the *Repast* convention, I wrote an agent class and a model class.

**The Agent Class:** The class constructor will take the type of agent (worker or firm) and a type of strategy (Random Walk, Highest Income, Lowest Income or Forward Looking), create an agent and place it in a random location.

There are two noteworthy methods: *pay* and *postStep*.

*pay* is called each run and will take the amount of money the agent earned (the wage for the worker or the profit for the firm) and will increase the agents stock of money by this amount.

*postStep* is called after each run and will, based on strategy, stock of money and wages in each location, send the agent to a new location.

**The Model Class:** Starts with setting up framework, locations and listeners for the diagrams. Next, all the agents (workers and firms) are created and added to a list of agents (*agentList*).

The Repast framework will call the *step* method for each step of the simulation. One simulation may run for 3000 steps or more.

The *step* method walks thru the *agentList* and find out where each agent is. Based on their distribution wages and profits are allocated for each location. Those wages and profits will than be paid to the agents calling the *pay* method.

Finally the framework automatically calls the *postStep* method, notifies all listeners to update the diagrams with the current values and gets ready for the next step.

**A run with one strategy: *Random Walk*.** Every period the agents get paid. As soon as an Agent stocks up enough money to move (in this case the

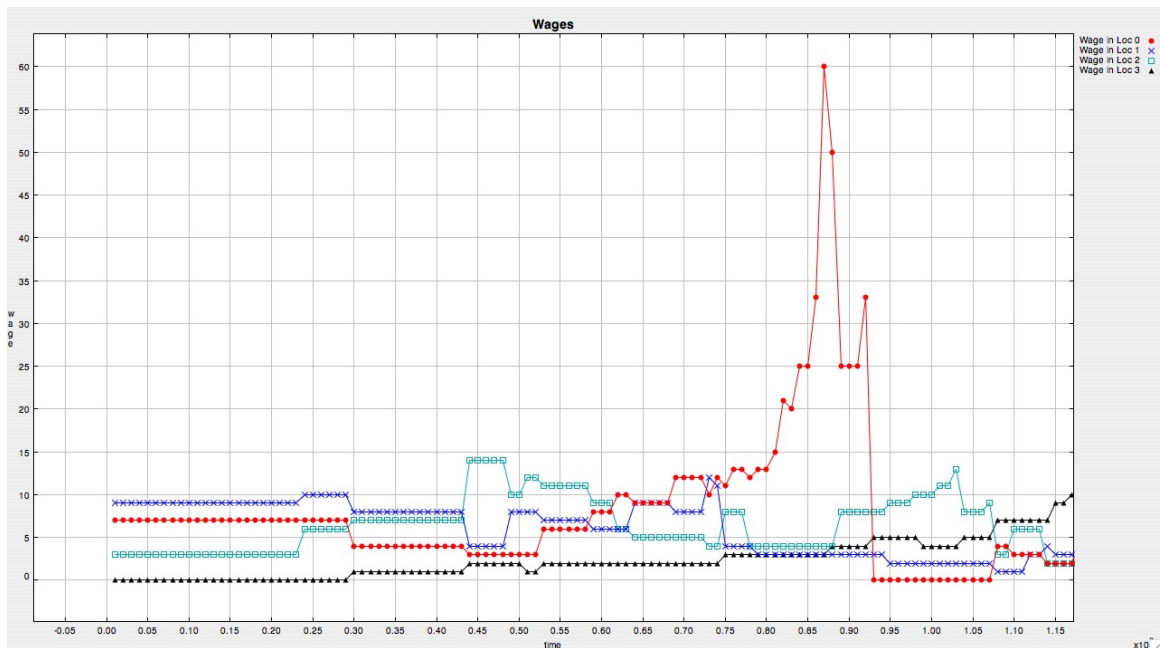


Figure 1: Random Walk

travel costs are set to 200) he will randomly choose a new location. Which will set the agents stock back by 200.

The resulting wage distribution is shown in Figure 1.

The high pick at period 87 comes from the fact that workers in location zero (red) have the highest wages, which allows them to move quickly and they will, since their strategy is a Random Walk. Each worker moving away from location zero will future increase wages in zero, until (randomly) a firm will move away from location zero and by doing so, bringing wages down again.

**A run with all four strategies evenly spread over 210 agents:** If we look at the *Number of Firms* in Figure 2 we can see that there is one firm in location zero, four in location one, two in location two and three in location three. Initially the firms earn nothing since not enough workers are in each location for a firm to break even (see equation 4). The 50 workers in location zero earn the lowest wage ( $1/50$ ).

Workers following the *Lowest Income Strategy* (see below) keep moving to location zero. At some point the firm in location zero breaks into profit, but diminishing returns on employees only gives a small increase in profit for each additional worker.

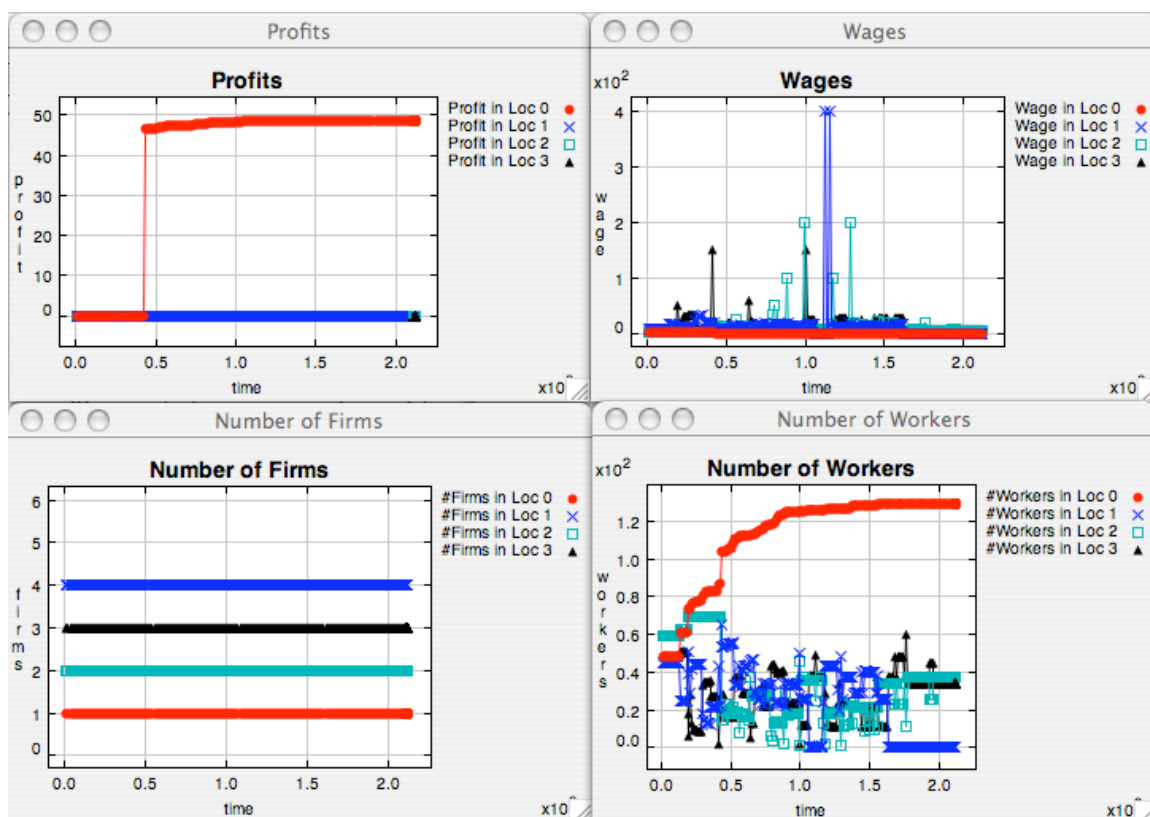


Figure 2: A run with all four strategies evenly spread over 210 agents

The other strategies result in a pseudo random behavior as can be seen in the *Number of Workers* graph.

**Random Walk Strategy** An agent randomly changes his location if he can afford to do so.

**Highest Income Strategy** Comparing all wages, the agent moves to the location with the highest wage, if he can afford to do so.

**Lowest Income Strategy** As above but following the lowest wage.

**Forward Looking Strategy** This is supposed to be the smart strategy. The agent looks for the location with the highest wage and compares his lifetime income<sup>3</sup> with the lifetime income in the new location minus travel costs (the agent assumes the current wages stay constant).

<sup>3</sup>The calculation of the lifetime income is simplified with a discount factor of  $\beta = 1$ .



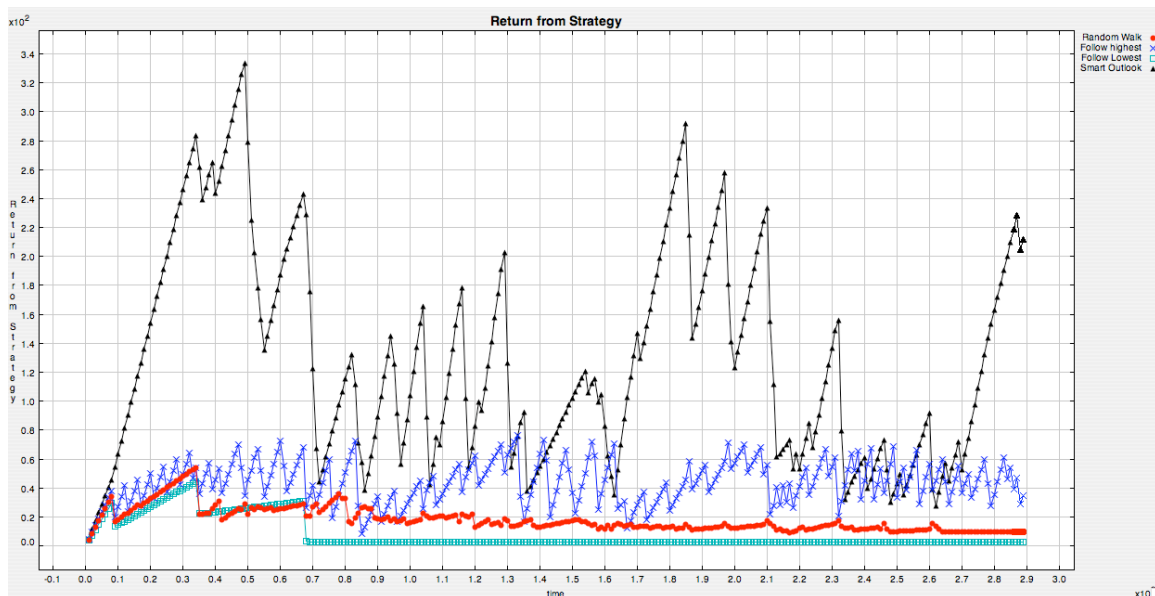


Figure 3: The average return on each strategy

The *Forward Looking Strategy* (black) was the dominant strategy in 20 separate runs of the simulation. Followed by the *Highest Income Strategy* (blue). The *Random Walk Strategy* (red) and the *Lowest Income strategy* (green) usually end up close to zero.

**Stability:** Occasionally the simulation gives a self repeating result (Figure 4). With a constant return for the *Forward Looking Strategy*, a self repeating step function for the *Highest Income Strategy*, a zero and none result *Lowest Income strategy* and a slightly positive *Random Walk Strategy*. This behavior will not change even for long runs ( $t = 3000$ ). It forms a quasi stable result.

## 6 Reflection

### Problems with the simulation:

- *Complexity:*  $v$  variables (number of workers and firms in each locations, wages of workers, profits of firms, gross product of locations, return of strategies,...) in  $l$  locations, with  $a$  agents,  $s$  strategies, during  $t$  time

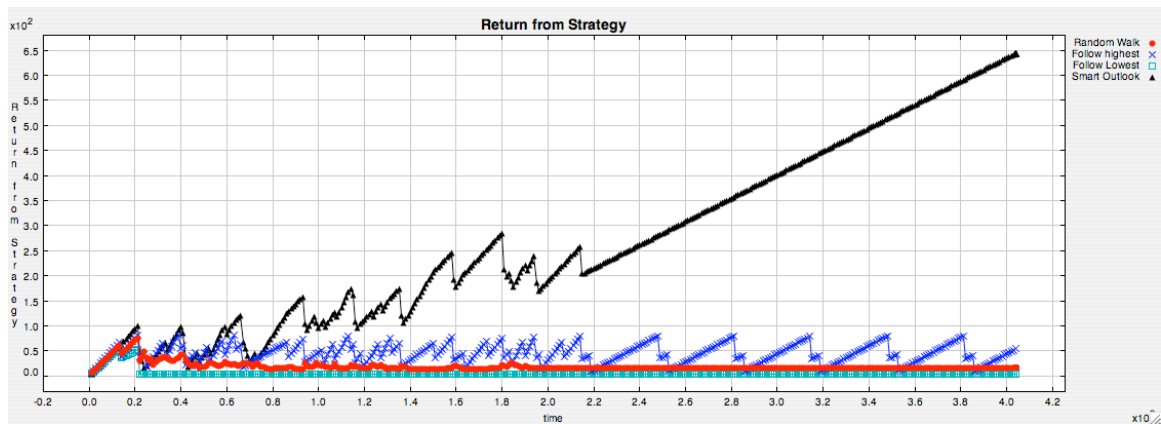


Figure 4: Constantly self repeating result

slots and  $r$  runs of the simulation. That gives  $v * l * a * s * t * r$  values to monitor<sup>4</sup>.

Repast has good visualization tools (see graphs above), but one still has to work thru a number of complex graphs for each simulation.

Complexity quickly becomes a problem. Even for such a small simulation.

- *Parameters*: The simulation is based on external parameters. Travel costs, life expectation, initial distribution of agents, strategies,...

One is tempted to "design" the parameters so they will produce the results one is looking for. If the results are unrealistic, it might be easier to fiddle with the parameters than reanalyze and possibly rewrite the model.

It's like choosing the assumption afterwards to produce the expected result. This is a problem one has to constantly keep in mind.

### Possible extensions:

- An important further step would be to deal with with complexity. Better visualization might help. The movement of workers could we visualized on a map. With different symbols for workers and firms. Repast offers tools to visuals agents on a grid. Therefor should the development of a graphical extension for the model be straight forward.

<sup>4</sup>Even a minimal setup like  $v = 6, l = 4, a = 210, s = 4, t = 2000, r = 30$  yields more than a billion values.

- Once complexity is reduced, more advanced strategies can be introduced.

### **Evolutionary approach:**

- If we let agents prosper and die we can evolve strategies. Based on the knowledge base (wages in various countries now and in past timeframes) we can mutate strategies and possibly find better (fitter) strategies to pick the right location for the agent.

Better strategies will survive and propagate.

### **Using the model to test assumptions:**

- Locations with high immigration costs for firms -because of bad infrastructure, great distance or political instability- may never attract firms (no matter how low wages are). This will keep wages at a low state. Possibly down to a state where workers can not afford to emigrate. Irresistibly keeping wages down and workers in poverty.
- With low costs of migration, I expect homogenization of wages. Once agents have saved enough they will start to migrate freely. Workers with low wages will attract firms -pushing up wages- and allowing workers to emigrate. This should lead over time to a shaky but roughly constant homogenize wage level for all locations.

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